Numerical modeling of the prestressing losses in prestressed concrete beams by modal analysis method

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Abstract: In this paper, a numerical modal analysis method is carried out to investigate the effect of cable strain loss on the natural frequencies in prestressed concrete beams. A simply supported concrete beam is modeled using the commercial finite element code ABAQUS. The beam is composed of concrete and prestressing strands. Firstly, a general geometrically nonlinear static analysis is carried out on the beam in order to obtain the camber deflection under different prestressing magnitudes. The obtained equilibrium configuration is then used to perform a linear and nonlinear modal analysis. Thereby, a three dimensional finite element C3D8 is used for the concrete in combination with a damaged plasticity model (CDP) are considered. Whereas, the prestressing strands are modeled with an embedded T2D2 truss element. The obtained results show a good agreement with previous numerical and experimental works in literature. Furthermore, the study showed that the safety of prestressed beams depend mainly on the level of prestressing load.

1. Introduction

Prestressed concrete structures are often used in construction for their performance compared to those built with ordinary reinforced concrete, especially in bridge structures. The identification of tension level in prestressed concrete beams for both the conceptual phase and service state is an important endeavor. In order to evaluate their structural characteristics, the modal analysis appears to be one of the most adopted methods for damage detection in structures. The safety of this type of design depends on the safety of its prestressing tendons. According to the literature, several studies have been conducted to study the effect of prestressing forces on the vibration frequency variation of the prestressed concrete beams. These studies have shown several contradictory argumentations regarding the influence of the prestressing on the vibration behavior.

Among these studies, one can cite the work of Clough (1975). The author deduced an analytical equilibrium equation for a beam axially loaded. It has been concluded that the eigenfrequencies of the beam decrease when the axial compressive load increases. Starting from this equation, Saiidi and Douglas (1994) elaborated an experimental test on a prestressed concrete beam. The authors found that the increase in prestressing forces leads to an increase in the natural bending frequencies. Thereafter, this work has been discussed by (Dall'asta and Leoni 1996; Jain and Goel 1996). The authors concluded that, in case of the axial prestressing forces are supposed as external forces, the

compression softening effect of the concrete could be responsible for the decrease in natural bending frequencies. In 2004, Kim et al. (Kim et al. 2004) used an empirical mathematical formulation to study the effect of prestressing loads on natural frequencies. The authors found that the natural frequencies increase when the prestressing forces are increased. The same results were also found using the finite element model (Law and Lu 2007; Bruggi et al. 2008; Breccolotti et al. 2009) as well as experimentally (Noh et al. 2015). The latter investigated the effects of several parameters such as: prestressing load, eccentricity and tendon profile. On the other hand, Hamed and Frostig (2006) developed a nonlinear analytical model of a post-tensioned beam. It has been found that the prestress force magnitude has no effect on the vibration of the natural bending frequencies.

In this paper a numerical modeling analysis is performed to investigate the prestressing levels effect on the variability of natural frequencies in a prestressed concrete beam. The embedded element formulation is used to model the interaction between steel and concrete. Furthermore, the concrete damaged plasticity model is adopted for the nonlinear behavior analysis of the concrete beam. The latter permit damage detection in concrete following the critical stress states of the prestressed concrete beam. Thereafter, a modal analysis is carried out to find a satisfactory compromise between the aforementioned contradictions found in the literature.

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2. Mechanical Constitutive Theories

2.1 Non-damaged concrete constitutive laws

The stress-strain behavior is taken according to Eurocode2 (2004). The secant elasticity modulus is:

$$E_{cm} = 22000 \left[\frac{f_{cm}}{10} \right]^{0.3} \tag{1}$$

The stress-strain relation for nonlinear structural analysis of the concrete is given by equation (2), where σ_c is the compressive stress in the concrete and f_{cm} is the compressive strength

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k-2)\eta} \tag{2}$$

Where,

$\eta = (\varepsilon_c / \varepsilon_{c1})$, $\varepsilon_{c1} = 0.0024$ for $f_{ck} = 45\,MPa$, $f_{cm} = f_{ck} + 8MPa$,

$k = 1.05 \times \text{Ecm} \left(|\varepsilon_{c1}| / f_{cm} \right)$, ε_c is the compressive strain in concrete, ε_{c1} is the strain at peak stress.

2.2 Damaged concrete constitutive laws

The concrete damaged plasticity (CDP) model is used as a constitutive model which is highly recommended for the nonlinear analysis of concrete structures (Arab et al. 2011). It appears in the literature that Lee and Fenves (1998) were the first who developed the CDP model. Figure 1 illustrates the Stress-Strain relation of the damaged plasticity model in both tension and compression, respectively. Furthermore, Table 1 gathers the different stresses and strain relations that can be deduced from Figure 1 where the subscripts "t" and "c" stand for tension and compression, respectively.
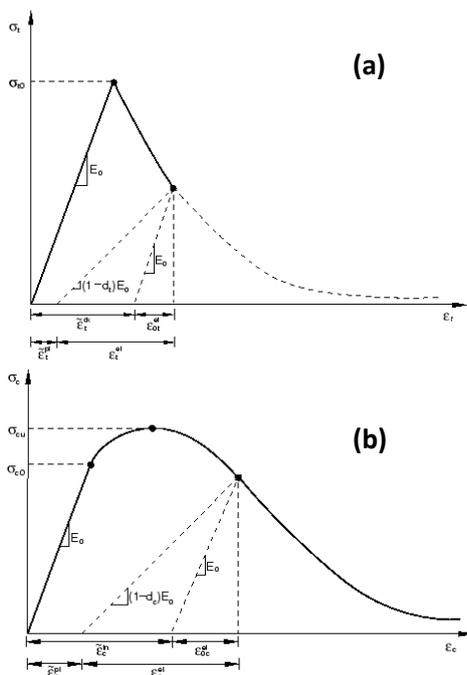

Fig 1. Stress-strain relation of the damaged plasticity model in: (a) tension and (b) compression (Abaqus analysis user's guide (2014)).

Table 1. Stresses and Strains relations.

Tension (figure 1a)	Compression (figure 1b)
$\sigma_t = (1 - d_t) E_0 \left(\varepsilon_t - \tilde{\varepsilon}_t^{pl} \right)$	$\sigma_c = (1 - d_c) E_0 \left(\varepsilon_c - \tilde{\varepsilon}_c^{pl} \right)$
$\tilde{\varepsilon}_t^{pl} = \tilde{\varepsilon}_t^{ck} - \left[\frac{(d_t \times \sigma_t)}{(1 - d_t)} \times E_0 \right]$	$\tilde{\varepsilon}_c^{pl} = \tilde{\varepsilon}_c^{in} - \left[\frac{(d_c \times \sigma_c)}{(1 - d_c)} \times E_0 \right]$
$\tilde{\varepsilon}_t^{ck} = \varepsilon_t - \varepsilon_{0t}^{el}$	$\tilde{\varepsilon}_c^{in} = \varepsilon_c - \varepsilon_{0c}^{el}$
$\varepsilon_{0t}^{el} = \sigma_t / E_0$	$\varepsilon_{0c}^{el} = \sigma_c / E_0$

where σ_t and σ_c are the stresses, d_t and d_c are damages, E_0 is the elastic modulus at the initial state, ε_t and ε_c are the total strains, $\tilde{\varepsilon}_t^{pl}$ and $\tilde{\varepsilon}_c^{pl}$ are the effective plastic strains, $\tilde{\varepsilon}_t^{ck}$ and $\tilde{\varepsilon}_c^{ck}$ are the effective cracking strains, ε_{0t}^{el} and ε_{0c}^{el} are the elastic strains.

2.3 Damage evolution law

To evaluate the stiffness degradation, the damage evolution law is considered, which depends on compression and tension behavior (Equation (3)). The mechanical parameters of steel, concrete and damaged plasticity are taken as given in Table 2.

$$\begin{cases} d_c = 1 - e^{\left(-a_c \varepsilon_c^{in} \right)} d_c = 1 - (\sigma_c / f_{cm}) \text{ For } (\tilde{\varepsilon}_c^{in} - \varepsilon_{0c}^{el}) > \varepsilon_c \\ d_t = 1 - (\sigma_t / f_{tm}) \text{ For } \varepsilon_t > 0 \end{cases} \tag{3}$$

3. Finite element modeling

3.1 Studied structure

In this study, a simply-supported prestressed concrete beam is considered. The beam has a length L=600 cm with a cross sectional area of (15x30) cm² (Figure 2). The prestressing tendon is eccentric from the neutral axis bye=10cm.

Table 2. Mechanical Parameters of steel, concrete and plastic damaged concrete.

Concrete plasticity parameters	Value
Dilation angle (ψ)	35°
Eccentricity	0.1
f_{b0}/f_{c0}	1.16
$k = (\rho_t / \rho_c)$	0.667
Viscosity parameter	0.001
Concrete mechanical parameters	Value
Compressive strength f_{cm}	45MPa
Tensile strength f_{tm}	4.55MPa
Young modulus E_c	34290MPa
Density ρ	2400Kg/m³
Poisson's ratio v	0.18
Steel mechanical parameters	Value
Young modulus E_p	209000 MPa
Density ρ	7850Kg/m³
Poisson's ratio v	0.3

f_{b0} / f_{c0} : Ratio of initial equi-biaxial and initial uniaxial compressive yield stresses. k : Deviatoric failure stress surface of concrete.
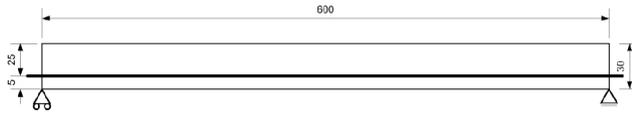
Fig 2. Prestressed concrete beam (Bruggi et al. 2008).

The tendon is made of three wire strands with a nominal diameter φ=1.5 cm and a cross sectional area A=1.40 cm². It should be noted that the ordinary reinforcement is neglected for the sake of simplicity.

3.2 Finite element mesh

The numerical modeling is performed using finite element ABAQUS software. The concrete is modeled using a linear brick element C3D8, which is a timely cost effective element. The tendon is represented with a truss element T2D2, which act purely in the longitudinal direction. A uniformly meshing size was considered with 5520 elements in the whole model. The interaction properties between the prestressing tendon and the concrete are established using embedded element technique. This type of constraint gives a rigid connection between contiguous nodes and assures a good compatibility between different types of finite elements (truss/solid) (figure 3).

3.3 Passive case study (non-prestressed tendon)

The First step is the numerical modeling of the simply supported beam without strands. The second step is by adding non-prestressed strands to investigate their effect on the natural bending frequencies. The aim of this analysis is to study the effect of tendons number, on the natural bending frequencies.

3.4 Prestress application

In this case, a technique is carried out under several steps. The first one is a general static analysis created to permit the release of prestressing forces on the concrete beam, which was introduced as initial stresses in the tendon (equation (4)).

$$\sigma_{xx} = f_{xx} / A_p \qquad (4)$$

Where, σ_{xx} is the longitudinal stress in strands, f_{xx} is the applied force level on strands and A_p is the cross sectional area of strands. The sequential applied forces in the tendon are f_{xx} = 22.5, 45, 67.5, 90 and 180 KN for one strand (Bruggi et al. 2008). As aforementioned, the tendon is made of three wire strands; it is modeled as one strand with its equivalent area.
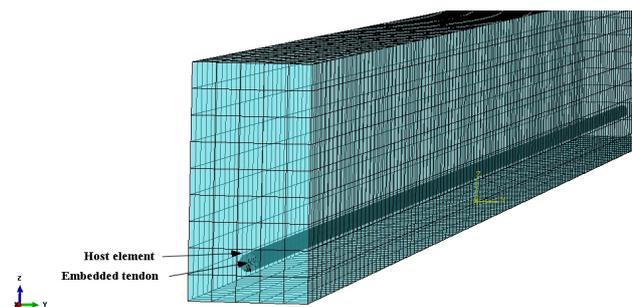

Fig 3. Finite element details of the embedded profile inside the host element.

The second step is also a general static analysis created to take into account the gravitational loads. The geometric nonlinearity is activated in order to obtain equilibrium between the prestressing magnitudes and the beam's self-weight. The last step is the eigenfrequencies extraction, where it depends on the camber deflection of the previous static analysis.

4. Results and discussion

4.1 Non-prestressed beam

In this study, the non-prestressed case is considered in order to validate the numerical model. The table 3 gathers the results found with the present model and those of Bruggi et al. (2008). From table 2, it be seen that the obtained results show a good agreement with those found in the literature. Moreover, the results showed that the number of passive tendons embedded inside the beam enhances the vibration frequencies, specifically in the first mode with 2% for the no strands-three strands configurations.

4.2 Prestressed beam

4.2.1 Linear analysis

Before modal analysis, it is important to achieve camber deflection during static analysis. Once the stresses are transferred, the beam starts to feel its own weight due to gravity application. The figure 4 shows the effect of prestressing magnitude on the camber deflection of the beam at mid-span. From figure 4, it can be observed that the deflection values depend on the applied prestressing magnitudes. Furthermore, the eccentricity of the tendon, which generates a negative moment of inertia, significantly affects the elastic deformations. The obtained results show a small value of deformation compared to the beam's length, a deflection of 20 mm for the highest value of prestressing (figure 4).

Table. 3. Effect of the non-prestressed strands on modal frequencies

| Strands N°	Mode 1(Hz)		Mode 2 (Hz)		Mode 3 (Hz)	
	Present study	Bruggi (2008)	Present study	Bruggi (2008)	Present study	Bruggi (2008)
0	14.182	14.141	54.615	54.553	105.84	106.90
1	14.284	14.312	55.021	54.571	106.60	107.53
2	14.378	14.471	55.392	54.563	107.29	108.08
3	14.463	14.619	55.730	54.567	107.93	108.62
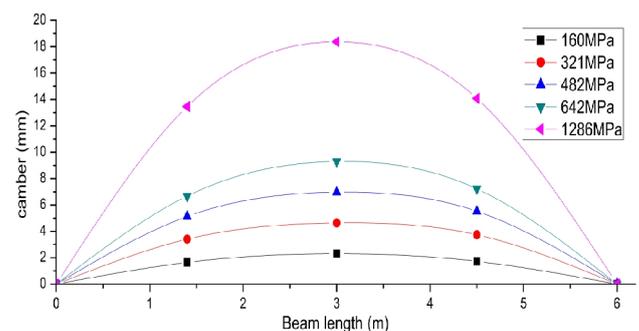
Fig 4. Finite element details of the embedded profile inside the host element.

Nevertheless, these small deflections could generate excessive stress in critical sections at mid-span. Indeed, the applied prestressing forces in this case deal with a quasi-brittle material that has a weak ultimate tensile stress. Figure 5 illustrates the results of the effect of prestressing on the longitudinal S_{11}stress of the beam. Three levels of prestressing values were considered, namely: 0 MPa, 160 MPa and 1286 MPa. In the case of 0 MPa, it is observed that the beam generates tensile stresses at the bottom flange and compressive stress at the upper flange. The latter result could be explained by the fact that the beam is acting only against its self-weight. Moreover, the maximum tensile stress observed is about σ_t = 2.011 MPa, which is smaller than the ultimate tensile strength of the used concrete (f_{tm}=4.55 MPa) given in table 2. In the light of the above, one can conclude that the beam is able to carry its own weight without damage expectations. In the case of 160 MPa prestressing force, the static results show that the tensile stresses disappear in lower flange and a very small value of tensile stresses has occurred in the upper flange. Regarding the case of 1286 MPa prestressing force, the beam generates a tensile stress on the upper flange σ_t =9.136 MPa. In this case, the stress overpasses the ultimate tensile strength of the used concrete f_{tm}= 4.55 MPa (table 2). The latter result gives the necessity to conduct a nonlinear analysis.

Table 4 exhibits the relationship between prestress forces and their corresponding frequencies of the beam. From table 4, it can be observed that frequencies follow linear augmentation. Indeed, the frequencies have shifted from 14.463 Hz in the case of a non-prestressed beam to 14.469 Hz in the case of maximum prestressed magnitude. In the second mode, it can be seen that the natural bending frequencies have decreased, which can be explained by the use of the isostatic boundary condition. In the third mode, one can observe an augmentation of 0.14 Hz and this is due to enhancing of the flexural stiffness of the global structure.

4.2.2 Non-linear analysis

Figure 6 shows the effect of the nonlinear stresses states in the half concrete beam. As in the linear analysis, three prestressing loads are considered in this section, namely: 0 MPa, 161 MPa and 1286 MPa.
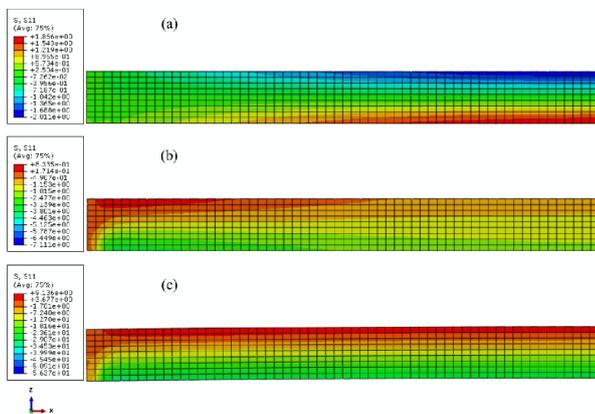

Fig 5. Linear stresses states in the half concrete beam with: (a) 0 MPa, (b) 161 MPa and (c) 1286 MPa.

Table. 4. Prestress force effect on the natural bending frequencies

| Prestress (MPa)	Mode1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)
0	14.463	55.730	107.930
160	14.464	55.723	107.950
321	14.464	55.715	107.960
482	14.465	55.708	107.980
642	14.466	55.700	108.000
1286	14.469	55.668	108.070
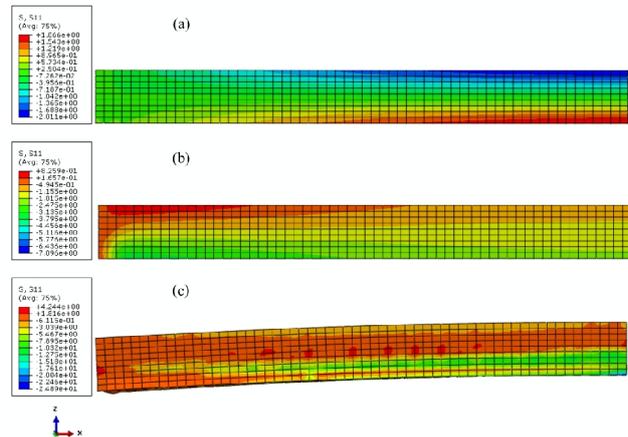
Fig 6. Nonlinear stresses states in the half concrete beam with:(a) 0 MPa, (b) 161 MPa and (c) 1286 MPa.

This could be explained by the fact that with these prestressing loads, no damage is expected to occur and the concrete stays in the elastic range. Regarding the case of 1286 MPa, it can be noticed that this load generates a maximum tensile stress at the upper flange of 4.24 MPa. This could be explained by the tensile plastic damage state undergoes by the beam as shown in figure 7.

Table 5 gives the results of the prestressing force effect on the vibration frequencies using the plastic damaged concrete model. From table 5, it can be well seen that the natural frequencies start to increase with the increasing the prestressing loads till 482 MPa, then decrease to reach the minimum with the 1286 MPa loads for the first and the third mode.

Table. 5. Prestress force effect on the natural bending frequencies.

Prestress (MPa)	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)
0	14.463	55.730	107.93
160	14.464	55.722	107.94
321	14.464	55.712	107.89
482	14.465	55.698	107.77
642	14.453	55.596	107.06
1286	10.442	35.340	87.864
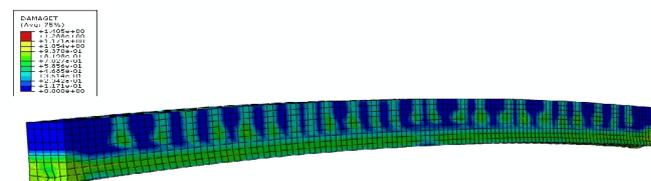
Fig 7. Tension damage state of the beam under the application of 1286 MPa of prestressing.

However, with the second mode the natural frequencies decrease with the increasing of the prestressing loads. The behavior noticed with the first and the third mode could be explained by the fact that the beam is loaded beyond the critical buckling load.

5. Conclusion

The aim of the present work was to investigate the effect of cable strain loss on the natural frequencies in prestressed concrete beams using the finite element method.

The initial linear analysis was focused on the effect of the number of passive non-prestressed strands on the natural frequency variation of the beam. The results show that changing the number of the embedded strands inside the beam affects positively the vibration behavior. This investigation gave encouraging results, since the real goal is to monitor the health of prestressed concrete beams.

Furthermore, in this work, a study is carried out on the effect of the prestressing load on the natural frequencies of a prestressed beam. The obtained results showed that a reduction in the pretension magnitude leads to a small reduction in frequencies. It was concluded that this is mainly due to the stiffness reduction of the beam, which depends on the safety of prestressing tendons.

The nonlinear analysis showed a small increase in the natural bending frequencies when applying small magnitudes of prestressing. However, for high level of magnitude that exceeds the critical buckling load, the natural frequencies significantly decrease. The latter observations are in good agreement with the nonlinear model of the concrete.

From this work we can conclude that the safety of prestressed beams depends on the level of prestressing load. Furthermore, the prestressing forces should not exceed the strength of the concrete in compression as well as the loss of prestressing should not generate a tensile stress in concrete.

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